Practice Exam 3
Mth 100

Polynomial Functions:

1. *Given* $f(x) = -3x + 7$ *and* $g(x) = 8x^3 - x^2 + 1$, *find* $(f-g)(2)$

$(f-g)(x) = -3x+7-8x^3+x^2-1$

$(f-g)(2) = -6+7-64+4-1 = -60$

2. The polynomial $7r^2 - 2r + 6$ describes the voltage in a circuit where r represents the resistance in the circuit.
   a. Find the voltage if the resistance is 6 ohms. 138
   b. Find the voltage if the resistance is 8 ohms. 438

Polynomial Division (use synthetic division, if possible):

3. *Divide* $\dfrac{14y^2 - 8y + 17}{7y+3}$

$$2y - 2 + \frac{23}{7y+3}$$

$7y+3 \;|\; 14y^2 - 8y + 17$

$\ominus 14y^2 \ominus 6y \downarrow$

$-14y + 17$

$\oplus -14y \oplus 6$

23

4. *Divide* $\dfrac{z^3 - z^2 + 3 - 11z}{z+2}$

$-2 \;|\; 1 \;\; -1 \;\; -11 \;\; 3$

$\quad\;\; -2 \;\; 6 \;\; 10$

$1 \;\; -3 \;\; -5 \;\; 13$

$$x^2 - 3x - 5 + \frac{13}{z+2}$$

Factor Completely:

5. $3x^3 - 6x^2 + 9x$
6. $g^3 + 64$
7. $2x^3y - 54yz^3$
8. $3t^2 + 2t + 1$
9. $z^2 - 49$
10. $a^2 - 24b^2 - 5ab$
11. $2q^2 - 13q - 24$
12. $(s+t)^2 - 2(s+t) + 1$
13. $16c^4 - 1$
14. $-x^2 - 3x + 28$
15. $xy + 2y + 4x + 8$

Practice Exam 3

Key

5) $3x^3 - 6x^2 + 9x$

$3x(x^2 - 2x + 3)$

6) $g^3 + 64$ ← sum of cubes.

$(g)^3 + (4)^3$

$(g+4)(g^2 - 4g + 16)$

7) $2x^3y - 54yz^3$

$2y(x^3 - 27z^3)$ ← diff. of cubes

$2y[x^3 - (3z)^3]$

$2y(x-3z)(x^2 + 3xz + 9z^2)$

8) $3t^2 + 2t + 1$

prime

9) $z^2 - 49$

$(z+7)(z-7)$

10) $a^2 - 24b^2 - 5ab$

$a^2 - 5ab - 24b^2$ 3, 8

$(a - 8b)(a + 3b)$

11) 1, 2

$2q^2 - 13q - 24$

$(2q + 3)(q - 8)$

12) $(s+t)^2 - 2(s+t) + 1$

Let $x = s+t$

$x^2 - 2x + 1$

$(x - 1)(x - 1)$

$(x-1)^2$

Substitute back

$(s+t-1)^2$

Practice Test 3 Mth 100

13) $16c^4 - 1$ ← diff of squares

$(4c^2)^2 - (1)^2$

$(4c^2+1)(4c^2-1)$ ← diff of squares

$(4c^2+1)((2c)^2-(1)^2)$

$\boxed{(4c^2+1)(2c+1)(2c-1)}$

14) $-x^2 - 3x + 28$

$-(x^2 + 3x - 28)$

$\boxed{-(x-4)(x+7)}$

15) $xy + 2y + 4x + 8$ group

$y(x+2) + 4(x+2)$

$\boxed{(y+4)(x+2)}$

16) $\frac{2q}{q-9}$

$\boxed{D = \{q \in \mathbb{R} \mid q \neq 9\}}$

all real numbers except 9

17) $\frac{w^2-16}{8-2w} = \frac{(w-4)(w+4)}{2(4-w)} = \frac{(w-4)(w+4)}{-2(w-4)} = \boxed{\frac{-w-4}{2}}$

18) $\frac{x^3-8}{x-1} \div \frac{x^2+2x+4}{3x^2-3x} =$

$\frac{x^3-8}{x-1} \cdot \frac{3x^2-3x}{x^2+2x+4}$ (flip)

$= \frac{(x-2)(x^2+2x+4)}{x-1} \cdot \frac{3x(x-1)}{x^2+2x+4}$ (factor)

$\frac{3x(x-2)}{1} = \boxed{3x^2-6x}$

19) $\frac{x+2}{x+5} - \frac{x-3}{x+7}$ common denom.

$= \frac{(x+2)(x+7)}{(x+5)(x+7)} - \frac{(x+5)(x-3)}{(x+5)(x+7)} =$

$\frac{x^2+9x+14-x^2-2x+15}{(x+5)(x+7)} = \boxed{\frac{7x+29}{(x+5)(x+7)}}$

20) $\frac{a^{-2}+b^{-1}}{a^{-1}-5b^{-3}} = \frac{1/a^2 + 1/b}{1/a - 5/b^3} = \left(\frac{a^2b^3}{1}\right)\left(\frac{\frac{1}{a^2}+\frac{1}{b}}{\frac{1}{a}-\frac{5}{b^3}}\right) = \frac{b^3+a^2b^2}{ab^3-5a^2} =$

$= \boxed{\frac{b^2(b+a^2)}{a(b^3-5a)}}$

Practice Exam 3 Key
Mth 100

Solve:

21) $x(x-5) = -6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x-2=0$ or $x-3=0$

$x=2$ $\quad$ $x=3$

22) $6y^2 - 5y = 4 - 28y$

$6y^2 + 23y - 4 = 0$

$(6y-1)(y+4) = 0$

$6y-1=0$ or $y+4=0$

$y = 1/6$ $\quad$ $y=-4$

23) $\dfrac{5}{x-3} = \dfrac{x}{x-2} + \dfrac{x}{x^2-5x+6}$

domain $x \neq 2, x \neq 3$

$\dfrac{5}{x-3} = \dfrac{x}{x-2} + \dfrac{x}{(x-3)(x-2)}$

$\dfrac{5(x-2)}{(x-3)(x-2)} = \dfrac{x(x-3)+x}{(x-3)(x-2)} = \dfrac{x^2-3x+x}{(x-3)(x-2)}$

$\dfrac{5x-10}{(x-3)(x-2)} = \dfrac{x^2-2x}{(x-3)(x-2)} \Rightarrow 5x-10 = x^2-2x$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x-2=0$ or $x-5=0$

~~$x=2$~~ reject $\quad$ $x=5$

24) $\frac{1}{2} = \frac{-3}{q+5}$ domain all $\mathbb{R}$'s except $q \neq 5$

$-6 = q + 5$

$q = -11$

25) $3 + \frac{9-2x}{8x} = \frac{5}{2x}$ (multiply every term by $8x$)

$24x + 9 - 2x = 20$

$\frac{22x}{22} = \frac{11}{22}$

$x = 1/2$

domain all $\mathbb{R}$'s so that $x \neq 0$